

4-kurs kechki FunkSIONal analizdan

Nazariy savollar

- 1) Chiziqli chegaralangan uzluksiz funksionallar. Ta'rif va misollar. Normallangan fazolarda berilgan funksionallar va ularning normasi
- 2) Chiziqli operatorlar. Chiziqli operatorning aniqlanish sohasi va qiymatlar sohasi. Uzluksizligi va chegaralanganligi. Operator normasi.
- 3) Chiziqli chegaralangan operatorlar fazosi. Operatorlar fazosida operatorlar ketma-ketligining kuchli (nuqtali) va tekis yaqinlashishi.
- 4) Qo'shma fazo Qo'shma operator. O'z-o'ziga qoshma operator. Misollar.
- 5) Operatorning spektri. Rezolventa. Xos sonlar va xos vektorlar. Misollar.
- 6) Chiziqli integral tenglamalar. Fredgolning 2-tur integral tenglamasi. Ajraluvchi yadroli integral tenglamalar.
- 7) Kompakt operatorlar. Chiziqli, chegarlanagan operatorning kompakt bo'lishining zaruriy va yetarli shartlari.
- 8) Teskari operatorlar. Teskarlanuvchi bo'lish sharti. Chiziqli operatorning teskari operatori bor bo'lishi haqidagi Banax teoremasi

№2

- 1) $(Ax)(t) = \int_0^2 t^2 e^s x(s) ds$ integral operatorning $C[0, 2]$ fazoda kompakt ekanligini isbotlang.
- 2) $(Ax)(t) = \int_{-1}^1 (1+ts)x(s) ds$ integral operatorning $C[-1, 1]$ fazoda kompakt ekanligini isbotlang.
- 3) $(Ax)(t) = \int_0^\pi \cos(t-s)x(s) ds$ integral operatorning $C[0, \pi]$ fazoda kompakt ekanligini isbotlang.
- 4) $(Ax)(t) = \int_0^\pi \cos(t+s)x(s) ds$ integral operatorning $C[0, \pi]$ fazoda kompakt ekanligini isbotlang.
- 5) $(Ax)(t) = \int_0^1 (t^3 s + s^2 t)x(s) ds$ integral operatorning $C[0, 1]$ fazoda kompakt ekanligini isbotlang.
- 6) $(Ax)(t) = \int_0^1 (st^2 + ts^2)x(s) ds$ integral operatorning $C[0, 1]$ fazoda kompakt ekanligini isbotlang.
- 7) $(Ax)(t) = \int_0^1 (t^2 s^2 + ts)x(s) ds$ integral operatorning $C[0, 1]$ fazoda kompakt ekanligini isbotlang.
- 8) $(Ax)(t) = \int_0^\pi \sin(t+s)x(s) ds$ integral operatorning $C[0, \pi]$ fazoda kompakt ekanligini isbotlang.

№3

- 1) $A: C[0; 1] \rightarrow C[0; 1]$, $(Ax)(t) = tx(0)$ operatorning diskret spektrini toping.

- 2) $A: C[0,1] \rightarrow C[0;1], (Ax)(t) = t^2 x(1)$ operatorning diskret spektrini toping
- 3) $A: C[0,1] \rightarrow C[0;1], (Ax)(t) = x(t)$ operatorning diskret spektrini toping.
- 4) $A: C[0,1] \rightarrow C[0;1], (Ax)(t) = tx(t)$ operatorning diskret spektrini toping.
- 5) $A: C[0,1] \rightarrow C[0;1], (Ax)(t) = x(1) + tx(0)$ operatorning diskret spektrini toping.
- 6) $A: C[0,1] \rightarrow C[0;1], (Ax)(t) = tx(0) + t^2 x(1)$ operatorning diskret spektrini toping.
- 7) $A: C[0,1] \rightarrow C[0;1], (Ax)(t) = t^2 x(t)$ operatorning diskret spektrini toping.

№4

- 1) $A \in L(X, X), X = R^2, Ax = (x_1 - x_2, x_1 + x_2)$ operatorning xos sonlari va xos vektorlarini toping..
- 2) $A \in L(X, X), X = R^2, Ax = (5x_1 + 3x_2, 2x_1)$ operatorning xos sonlari va xos vektorlarini toping..
- 3) $A \in L(X, X), X = R^2, Ax = (2x_1, 3x_1 + x_2 + 3x_3, 2x_3)$ operatorning xos sonlari va xos vektorlarini toping..
- 4) $A \in L(X, X), X = R^2, Ax = (x_1, x_2 + x_3, x_2 + x_3)$ operatorning xos sonlari va xos vektorlarini toping..
- 5) $A \in L(X, X), X = R^2, Ax = (2x_1 + x_2, x_1 + 2x_2)$ operatorning xos sonlari va xos vektorlarini toping.
- 6) $A \in L(X, X), X = R^2, Ax = (5x_1 - 3x_2, 2x_1)$ operatorning xos sonlari va xos vektorlarini toping.
- 7) $A \in L(X, X), X = R^2, Ax = (x_1 + 2x_2, 2x_1 + x_2)$ operatorning xos sonlari va xos vektorlarini toping.

№5

- 1) $A: R^3 \rightarrow R^3, Ax = (2x_1 + x_3, x_2 + 2x_3, x_1 + x_3)$ operatorga A^{-1} teskari operatorni toping.
- 2) $A: R^3 \rightarrow R^3, Ax = (x_1, x_1 - x_2, x_2 - x_3)$ operatorga A^{-1} teskari operatorni toping.
- 3) $A: R^3 \rightarrow R^3, Ax = (2x_1 + x_3, x_2 + 2x_3, x_1 + x_3)$ operatorga A^{-1} teskari operatorni toping.
- 4) $A: l_2 \rightarrow l_2, Ax = (x_1 + x_2, x_1 - x_2, x_3, x_4, \dots)$ operatorga A^{-1} teskari operatorni toping.

№6

- 1) $A: l_2 \rightarrow l_2, Ax = (x_1 - 2x_3, x_2 + 4x_3, x_3, x_4, \dots)$ operatorga qo'shma operatorni toping.
- 2) $A: L_2[0;1] \rightarrow L_2[0;1], Ax(t) = \int_0^1 \sin(t^2 s) x(s) ds$ operatorga qo'shma operatorni toping.
- 3) $A: L_2[0;1] \rightarrow L_2[0;1], Ax(t) = \int_0^1 \sin(ts) x(s) ds$ operatorga qo'shma operatorni toping.

4) $A: l_2 \rightarrow l_2$, $Ax = (x_2, x_1, x_3, x_4, \dots)$ operatorga qo'shma operatorni toping..

5) $A: l_2 \rightarrow l_2$, $Ax = (0, 0, x_1, x_2, \dots)$ operatorga qo'shma operatorni toping..

6) $A: l_2 \rightarrow l_2$, $Ax = (x_1 - x_2, x_1 + x_2, x_3, x_4, \dots)$ operatorga qo'shma operatorni toping..

№7

1) $\varphi(x) - 4 \int_0^{\pi/2} \sin^2 x \varphi(t) dt = 2x - \pi$ ajraluvchi yadroli integral tenglamani yeching.

2) $\varphi(t) = 2 \int_0^1 (1 + 3ts) \varphi(s) ds + t^2$ ajraluvchi yadroli integral tenglamani yeching.

3) $\varphi(t) = \int_0^{\pi/2} \sin t \cos s \varphi(s) ds + \sin t$ ajraluvchi yadroli integral tenglamani yeching.

4) $\varphi(x) + \int_0^1 x(e^{xt} - 1) \varphi(t) dt = e^x - x$ integral tenglamaning ushbu $\varphi(x) = 1$ funksiya yechimi ekanligini tekshiring.

5) $\varphi(t) = 2 \int_0^1 (1 + ts) \varphi(s) ds + t^2$ ajraluvchi yadroli integral tenglamani yeching.

6) $\varphi(t) = \int_0^{\pi/2} \sin t \cos s \varphi(s) ds + \sin t$ ajraluvchi yadroli integral tenglamani yeching

7) $\varphi(t) = -3 \int_0^{2\pi} \sin(t + s) \varphi(s) ds$ ajraluvchi yadroli integral tenglamani yeching

8) $\varphi(t) = 2 \int_0^1 (1 + ts) \varphi(s) ds + t^2$ ajraluvchi yadroli integral tenglamani yeching.

9) $\varphi(x) = \int_0^{\pi} \sin t \cos s \varphi(s) ds + \sin x$ integral tenglamaning ushbu $\varphi(x) = \sin x$ funksiya yechimi ekanligini tekshiring.

10) $\varphi(x) + 2 \int_0^1 e^{x-t} \varphi(t) dt = 2xe^x$ integral tenglamaning ushbu $\varphi(x) = 2e^x \left(x - \frac{1}{3} \right)$ funksiya yechimi ekanligini tekshiring.

11) $\varphi(x) + \int_0^1 x(e^{xt} - 1) \varphi(t) dt = e^x - x$ integral tenglamaning ushbu $\varphi(x) = 1$ funksiya yechimi ekanligini tekshiring.

№8.

1) $C[-2;2]$ fazoda berilgan $f(x) = x(2) + \int_{-2}^2 tx(t)dt$ funksionalning uzluksiz ekanligini ko'rsating.

2) $C[-2;1]$ fazoda berilgan $f(x) = 3x(-2) + x(0) - 5x(0,5)$ funksionalning uzluksiz ekanligini ko'rsating.

3) $C[0,1]$ fazoda berilgan $f(x) = \frac{x(0)}{2} + \frac{x(1)}{2}$ funksionalning chiziqli, uzluksiz ekanligini ko'rsating..

4) $(Ax)(t) = x(0) + tx(1)$, $A: C[0,1] \rightarrow C[0,1]$ operatorning chiziqli, chegaralangan ekanligini ko'rsating va normasini toping.

5) $(Ax)(t) = tx(t)$, $A: C[0,1] \rightarrow C[0,1]$ operatorning chiziqli, chegaralangan ekanligini ko'rsating va normasini toping.

6) $(Ax)(t) = x(0)t + x(1)t^2$, $A: C[-1,2] \rightarrow C[-1,2]$ operatorning chiziqli, chegaralangan ekanligini ko'rsating va normasini toping.

7) $(Ax)(t) = te^t x(t)$, $A: C[-2,2] \rightarrow C[-2,2]$ operatorning chiziqli, chegaralangan ekanligini ko'rsating va normasini toping..